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ФУНКЦИОНАЛЬНЫЕ СООТНОШЕНИЯ И ДИНАМИЧЕСКОЕ ПРЕДСТАВЛЕНИЕ ОБЪЕКТОВ

Кененбаев Э.
Институт математики НАН КР, Бишкек, Кыргызстан, elaman0527@gmail.com

***Аннотация**. Многие объекты, которые нужно представить на компьютере для научных и учебных целей, являются переменными. Для удобного алгоритмического представления автор предложил использовать функциональные соотношения – многоместные предикаты с участием наборов значений функций в различных точках. В Кыргызстане разрабатывается независимое интерактивное компьютерное представление естественных языков. Для этой цели Б.Баячорова и С.Карабаева предложили представить преобразующие глаголы «согнуть, выпрямить, разрезать, склеить». Такие глаголы - более сложные, чем глаголы, реализующиеся через «последовательность сдвигов»: «положить, взять, передвинуть». В данной статье построены математические модели некоторых глаголов и других понятий при помощи функциональных соотношений между точками виртуальных объектов. Количество точек в предикате показывает сложность понятия.*

***Ключевые слова:** переменный объект, язык, компьютерное представление, математическая модель, независимое представление*

ФУНКЦИОНАЛДЫК ӨЗ АРА КАТЫШТАР ЖАНА ОБЪЕКТТЕРДИ ДИНАМИКАЛЫК КӨРСӨТҮҮ

Кененбаев Э.
КР УИА Математика институту, Бишкек, Кыргызстан, elaman0527@gmail.com

***Аннотация**. Илимий жана окутуу максаттары үчүн компьютердекөрсөтүлүүчү көпобъект өзгөрүүчү болуп эсептелет. Алгоритмдик көрсөтүүнүнынңгайлуулугуүчүн автор функционалдык өз ара катыштарды - ар кандай чекиттерде функция маанилеринин топтомун камтыган көп орундуу предикаттарды колдонууну сунуш кылган. Кыргызстанда табигый тилдерди компьютерде көз карандысыз интерактивдик чагылдыруу иштелип чыгууда. Бул максат үчүн Б.Баячорова жана С.Карабаева "бүктө, түздө, кес, чапта" деген өзгөртүүчү этиштерди чагылдырууну сунуш кылган. Мындай этиштер "жылдыруулардын удаалаштыгы" аркылуу көрсөтүлүүчү "кой, ал, жылдыр" деген этиштерден татаалыраак. Бул макалада кээ бир этиштердин жана башка түшүнүктөрдүн математикалык моделдери элестетилген (виртуалдуу) объекттердин чекиттеринин арасындагы функционалдык өз ара байланыштар аркылуу курулду. Предикатта чекиттердин саны түшүнүктүн татаалдыгын көрсөтөт.*

***Ачкыч сөздөр:** өзгөрмө объект, тил, функционалдык өз ара катыштар, компьютердик сүрөттөлүш, математикалык модель, көз карандысыз сүрөттөлүш*

FUNCTIONAL RELATIONS AND DYNAMICAL PRESENTATION OF OBJECTS

Kenenbaev E.
Institute of Mathematics of NAS KR, Bishkek, Kyrgyzstan, elaman0527@gmail.com

***Annotation.** Many objects to be presented on computers for scientific and educational purposes are variable. For a convenient algorithmic representation, the author proposed using functional*

relations - multi-place predicates involving sets of function values at different points.Independent interactive computer presentation of natural languages is developed in Kyrgyzstan. For this purpose, B. Bayachorova and S. Karabaeva proposed to present transforming verbs "bend, straighten, cut, glue". Such verbs are more complex than ones implemented as "sequences of shifts": "put, take, move". In the paper mathematical models of some verbs and other notions are constructed by functional relations between points of virtual objects.
Keywords: *variable object, language, computer presentation, mathematical model, independent presentation*

Introduction

We consider controlled presentation of objects on a computer. Usually "Drag-and-Drop" operation is used for it. But many objects to be presented on computers for scientific and educational purposes are variable. For a convenient algorithmic representation, we [1] proposed using functional relations - multi-place predicates involving sets of function values at different points.

Independent interactive computer presentation of natural languages is developed in Kyrgyzstan. For this purpose, B. Bayachorova and S. Karabaeva[2] proposed to present transforming verbs "bend, straighten, cut, glue". Such verbs are more complex than ones implemented as "sequences of shifts": "put, take, move". In the paper mathematical models of some verbs and other notions are constructed by functional relationsbetween points of virtual objects.

Section 1 contains new, more general definition of functional relations.

Section 2 describes definitions of independent interactive computer presentation of natural languages.

Section 3 presents examples of notions defined by functional relations within the frames of independent interactive computer presentation of natural languages.

1. Functional relations

Denote $\boldsymbol{R}:=(-\infty,\infty)$, $\boldsymbol{R}_+:=[0,\infty)$.

Various tasks for differential equations can be presented in general form as follows. There are given (defined) a set F of functions $X\to Z$ (a general solution of the differential equation), a subset $X_0\subset X$ and a function $u_0:X_0\to Z$. Find such $u\in F$ that its restriction on the set X_0 coincides with the function X_0. The function u_0 presents initial, boundary or other types of conditions.

Also, various families of functions meet such construction too.

It may be generalized as follows. Denote the set of (non-empty) sets in $\boldsymbol{R}^2$as W.

Let $B\in W$, $f: B\times\boldsymbol{R}_+\to W$, $f(z,0)=z$. $f(B,t)$ are said to be *transformations* of B for $t>0$.

Let $z[1],\ldots, z[n]\in B$ be arbitrary or marked points, P be an n-place predicate.

If $P(f(z[1],t)\ldots, f(z[n],t))=$ *true* for all $t\geq 0$ then the family $\{f(B,t)|\ t\geq 0\}$ is said to be fulfilling the functional relation P.

Examples of functional relations.

1.1. Polynomials. The differential equation for a polynomial of n-th power is $u^{(n+1)}(x)=0$. The main functional relation is the Lagrange polynomial

$$u(x) = \sum_{k=1}^{n+1} \frac{(x-x[1])\dots(x-x[k-1])(x-x[k+1])\dots(x-x[n+1])}{(x[k]-x[1])\dots(x[k]-x[k-1])(x[k]-x[k+1])\dots(x[k]-x[n+1])} u(x[k]). \quad (1)$$

1.2. Solutions of ordinary linear differential equation. The first result on functional relation for values of solution in different points was obtained by J. De la Vallée Poussin (see [3] for example).

The equation

$$u^{(n)}(x)+p_1(x)\, u^{(n-1)}(x)+\dots+ p_n(x)\, u(x) = 0,\ \ a\le x\le b, p_k(x)\in C[a,b], \quad (2)$$

with conditions

$$u(x[i]) = c[i],\ i=1,\dots,\ n, \quad (3)$$

$c[i],\ i=1,\dots,\ n$ are real numbers, has a unique solution under the following restrictions on norms of coefficient-functions

$$||\, p_1||_{[a,b]}(b-a)+ ||\, p_2||_{[a,b]}\, (b-a)^2/2!+\dots + ||\, p_n||_{[a,b]}\, (b-a)^n/n! \ < 1. \quad (4)$$

1.3. Partial differential equations.

$$u_{xy}''(x,y)=0. \quad (5)$$

It is known that

$$u(x,y)+u(x+g,y+h)\equiv u(x+g,y)+u(x,y+h) \quad (6)$$

for solutions of the equation (6) for all $x,y\in\boldsymbol{R};\ g,\ h\in\boldsymbol{R}_{++}$.

Denote $z=(x,y)\in\boldsymbol{R}^2$; for (7)

$$z[1]= (x,y);\ z[2]=(x+g,y); z[3]=(x,y+h);\ z[4]=(x+g,y+h).$$

Here $Q=Q_2$ is the set of sets W of four points $\{z[1], z[2], z[3],z[4]\}$ arranged in form of coordinate rectangle.

2. Survey and definitions for independent presentation of objects

In 1878 M.Berlitz proposed a method with is considered as “the first-known immersive teaching method”. Such method was improved as “Total physical response” [4].

Winograd T. [5] proposed giving commands to a robot with such words as "table", "box", "block", "pyramid", "ball", "grasp", "moveto", "ungrasp".

Using these ideas, since [6] general methods for interactive computer presentations of natural languages are being developed [7-11]. If a computer presentation does not depend on the user’s knowledge and skills on similar objects then it is called *independent*. Such presentations are more effective because the user can learn a language inductively, and they begin thinking in it, without translation in mind.

The following definitions were proposed.

D e f i n i t i o n 1. If low energetic outer influences can cause sufficiently various reactions and changing of the inner state of the object (by means of inner energy of the object or of outer energy entering into object besides of such influences) at any time then such (permanently unstable) object is an *affectable object*, or a *subject*, and such outer influences are *commands.* A system of commands such that any subject can achieve desired efficiently various consequences from other one is a *language*.

D e f i n i t i o n 2. Let any "notion" (word of a language) be given. If an algorithm acting at a computer:performs (generating randomly) sufficiently large amount of situations covering all essential aspects of the "notion" to the user;gives a command involving this "notion" in each situation;perceives the user's actions and performs their results clearly on a display;detects whether a result fits the command,

then such algorithm is said to be a computer interactive presentation of the "notion".

Three techniques are proposed for the user's guessing:

The following definition is proposed by us on the base of Section 1.

D e f i n i t i o n 3. A marked set (2D-set on display) is a set *(S)* with some marked points on it $(x[1], x[2], x[3[\ldots \in S)$. Atransformable object is a (varying) set or union of marked sets, marked points of them are connected with *functional relations:* "$\zeta(x,y)$".

D e f i n i t i o n 4. Mathematical models consist of fixed (F_i) and movable and/or transformable(M_j) sets; an extended algorithm: temporal sequence of conditions of types

$(M_j \subset F_i)$, $(M_j \cap F_i = \varnothing)$, $(M_j \cap F_i \neq \varnothing)$; $\zeta(x_k(M_j), x_q(M_j))$.

D e f i n i t i o n 5. Transformation of objects can be presented by a controlled differential equation. Firstly, we consider motion of a flat object without inertia and not-self-moving objects. Let $S \in R^2$ be initial position of the object; $S(t)$ be the position and shape of the object at time t. We have the equation $y'(t,z)=F(t, S(t),u(t)),\ 0 \leq t < \infty$ with initial condition $y(0)= z \in S$, where $u(t)$ is a control function (given by the user), the function $F(s,u)$ (to be implemented by the programmer) is bounded, $y(t,z)\colon [0, \infty) \times S \to \mathbf{R}^2$ is the trajectory of the point $z \in S$. Let S and $S(t)$ be marked, defined by points $z[1], z[2],\ldots,z[k] \in S$ and their images correspondingly. Then $F=F(t,y[1], y[2],\ldots,y[k],u(t))$. Values $y[1], y[2],\ldots,y[k]$ are to be connected with functional relations. Computer interactive presentations are built on the base of mathematical models.

3. Examples of mathematical models for transformable objects

3.1. $k=2$, the functional relation is $dist(y[1], y[2])=const$. It can be used for verbs ROTATE, TURN; PULL.

3.2. $k=3$, the functional relations are $y[1]=const, y[2]=const, dist(y[2], y[3])=const$.

It can be used for the verb BEND.

3.3. $k>3$, the functional relations are

$dist(y[1], y[2]) = dist(y[2], y[3])=\ldots= dist(y[k-1], y[k]) = const.$

It can be used for the noun LENGTH, verb BEND.

3.4. $k \geq 2$. Firstly, two sets S_1 and S_2. $y[1] \in S_1$ and $y[2] \in S_2$. The functional relation is $y[1]= y[2]$ for any $t>t_1$. (Or more than two distinct objects).

It can be used for the verb GLUE.

3.5. $k=2$. The functional relations are $y[1]=const$, $dist(y[1], y[2])$ increases. The verb STRETCH.

Conclusion

We demonstrate that functional relations can be applied to enlarge the range of notions which can be presented within the frames of independent interactive computer presentation of natural languages. The minimal number of points in the predicate is an objective measure for complexity of the notion.

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